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PRESSURE DROP AND FILM HEIGHT MEASUREMENTS FOR ANNULAR GAS–LIQUID FLOW

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Abstract—Measurements of the pressure drop and the film height averaged around the circumference are presented for air and water flowing in horizontal 2.54 and 5.08 cm pipelines. Film height measurements are interpreted using relations similar to those that have been developed for vertical flows. The frictional pressure loss is found to be primarily related to properties of the liquid film and to be approximately independent of the amount of entrained liquid.

1. INTRODUCTION

Air and water flowing in a horizontal pipeline attain an annular flow configuration at high gas velocities. A fraction, $E = W_{LE}/W_L$, of the mass flow of liquid, W_L, is entrained as droplets in the gas. The remainder flows as a liquid film along the wall with a mass flow rate of $W_{LF} = W_L - W_{LE}$. Because of gravitational effects the film is not distributed uniformly around the circumference but is thicker at the bottom of the pipe.

This article describes measurements of film height and pressure drop for fully developed air–water flow in horizontal 2.54 cm and 5.08 cm pipelines. In theses by two of the authors (Dallman 1978; Laurinat 1982) the following correlation was presented for E for air water flows:

$$\frac{E}{1-\dfrac{W_{LFC}}{W_L}} = \frac{3.6\times10^{-8}[(d_t-2m)\rho_G^{1/2}\rho_L^{1/2}U_G^{3}]^{1.5}}{1+3.6\times10^{-8}[(d_t-2m)\rho_G^{1/2}\rho_L^{1/2}U_G^{3}]^{1.5}}. \quad [1]$$

Here 3.6×10^{-8} is a dimensional constant with units of $(\sec^3/\mathrm{kg\,m})^{1.5}$; (d_t-2m), the mean diameter of the core; ρ_L, the liquid density; ρ_G, the gas density; and W_{LFC}, the critical film flow rate.

Equation [1] shows that the entrainment increases rapidly with gas velocity, U_G, until at high velocities a fully entrained condition is reached, $E = 1 - W_{LFC}/W_L$, for which further increases in gas velocity will not cause changes in the film flow rate.

Our principal goals in this paper are to show the effect of pipe diameter and fluid flow rates on pressure drop and film height, and to explore whether the increase in the frictional pressure losses over what would be obtained if gas were flowing alone can be associated primarily with flow properties of the annular film.

Measurements of entrainment and pressure drop for upward flow of air and water in a vertical pipeline have been obtained by Willis (1965), Whalley *et al.* (1973), Gill *et al.* (1963), and by Collier & Hewitt (1961). Henstock & Hanratty (1976) have shown that these results support the notion that the ratio of the measured friction factor, f_I, to the friction factor for gas flow in a smooth pipe, f_s, is a function of the ratio of the average height of the film to the tube diameter, m/d_t, and that this function is independent of the amount of liquid entrained in the gas. They found that the following simple relation can be used to correlate the data for upflow:

$$\frac{f_I}{f_s} = 1 + 1400F. \qquad [2]$$

Here F is a film flow factor analogous to the Martinelli flow factor, defined by [13]. It differs from the Martinelli parameter in that it is defined using the liquid flow in the film, W_{LF}, and not the total liquid flow, W_L, and it does not require different expressions for laminar and turbulent film flows.

Butterworth (1973) and Swanson (1966) have presented measurements of film height, entrainment and pressure drop for air and water flowing in horizontal 3.18 and 2.54 cm pipelines. However, as pointed out by Henstock & Hanratty (1976), these do not cover a wide enough range of conditions to come to any definite conclusions regarding the influence of the pipe diameter and the flow rates of the gas and liquid.

2. CORRELATION OF RESULTS FOR VERTICAL FLOWS

(a) *Film height*

For a film which is uniformly distributed about the circumference of a pipe of radius, R, the variation of the stress, τ, with radial location, r, is given by

$$r\tau = R\tau_w - \frac{\rho_L \mathbf{g} R^2}{2} + \frac{\rho_L \mathbf{g} r^2}{2}, \qquad [3]$$

where

$$\mathbf{g} = \frac{1}{\rho_L}\left|\frac{\mathrm{d}p}{\mathrm{d}x}\right| + g \qquad [4]$$

for upflow,

$$\mathbf{g} = \frac{1}{\rho_L}\left|\frac{\mathrm{d}p}{\mathrm{d}x}\right| - g \qquad [5]$$

for downflow, and

$$\mathbf{g} = \frac{1}{\rho_L}\left|\frac{\mathrm{d}p}{\mathrm{d}x}\right| \qquad [6]$$

for horizontal flow. As suggested by Henstock & Hanratty (1976), a characteristic stress, τ_c, can be defined for the film as

$$\tau_c = \tau_w\left(1 - \frac{2}{3}\frac{m}{d_t}\right) - \frac{1}{3}\rho_L \mathbf{g} m, \qquad [7]$$

which, for $m/d_t \to 0$, is given as

$$\tau_c = \frac{2}{3}\tau_w + \frac{1}{3}\tau_1, \qquad [8]$$

with τ_w being the stress at the wall and τ_I, the stress at the interface. A dimensionless film height is defined as $m^+ = mv^*/\nu_L$, where the friction velocity is $v^* = (\tau_c/\rho)^{1/2}$. The velocity profile in the film can be calculated for laminar flows by relating τ in [3] to the velocity gradient by Newton's law of viscosity. The integration of this profile yields the following relation for the volumetric flow in the film, Q:

$$\mathrm{Re}_{LF} = 2m^{+2}, \qquad [9]$$

with $\mathrm{Re}_{LF} = 4Q/P\,\nu_L$. It is noted that because of the use of a characteristic stress, τ_c, the dependency on **g** does not appear directly in [9].

A differential equation describing the variation of fluid velocity for a turbulent flow can be obtained if it is assumed that the stress is related to the velocity gradient by law of wall equations developed for turbulent single phase flows. The integration of this differential equation yields

$$m^+ = f\left(\mathrm{Re}_{LF}, \alpha m^+, \frac{m}{d_t}\right), \qquad [10]$$

which, as pointed out by Henstock & Hanratty (1976), is insensitive to the particular velocity profile equation that is chosen. The term m/d_t enters because of the curvilinear coordinate system needed to describe a pipe flow. Usually, in annular flow m/d_t is small enough that its influence can be neglected. The term $\alpha m^+ = -\mathbf{g}m/v^{*2}$ characterizes the stress distribution in the film. For horizontal flows, vertical upflows or vertical downflows at high gas velocities αm^+ is also small enough that its influence can be ignored and the following relation is derived (Henstock & Hanratty 1976):

$$m^+ = \gamma\,(\mathrm{Re}_{LF}) = [(0.707\,\mathrm{Re}_{LF}^{0.5})^{2.5} + (0.0379\,\mathrm{Re}_{LF}^{0.90})^{2.5}]^{0.40} \qquad [11]$$

Equation [11] is in approximate agreement with available air–water data for upflows.

In order to calculate m from [11], it is necessary to be able to evaluate the characteristic stress, τ_c. This, in turn, requires a method for calculating τ_I. If [2] is used to evaluate τ_c the following relation of film height to controlled variables is obtained:

$$\frac{m}{d_t} = \frac{6.59}{(1 + 1400F)^{1/2}}, \qquad [12]$$

with the flow factor F defined as

$$F = \frac{\gamma\,(\mathrm{Re}_{LF})}{\mathrm{Re}_G^{0.90}} \frac{\nu_L}{\nu_G} \left(\frac{\rho_L}{\rho_G}\right)^{1/2}. \qquad [13]$$

(b) *The friction factor*

The interfacial stress can be related to the gas velocity, U_G, by the equation

$$\tau_I = \frac{1}{2}\rho_G U_G^2 f_I, \qquad [14]$$

where f_I is the interfacial friction factor. For gas flowing through a smooth pipe f_I equals f_s, where

$$f_s = 0.046\,\mathrm{Re}_G^{-0.20}. \qquad [15]$$

Here, Re_G is defined by using the diameter of the gas space, $d_t - 2m$. (However, since m/d_t

is usually small for annular flows, we have evaluated it using d_t in all of the results presented in this paper.)

It is argued that the ratio f_I/f_s is larger than unity in annular flow because waves on the film have the same effect as a roughened wall. If the sizes of the waves on the film scale within the film height then one can assume $f_I/f_s \sim m/d_t$. It follows from [12] that $f_I/f_s \sim F$. Henstock & Hanratty (1976) found for vertical flows that

$$\frac{f_I}{f_s} = f\left(F, \left|\frac{\tau_I}{\rho_L g m}\right|\right). \tag{16}$$

The term $|\tau_I/\rho_L g m|$ represents the relative forces due to interfacial drag and gravity. Its influence on f_I/f_s can be interpreted as due to a change of the characteristics of the wave pattern. For large gas velocities or large values of $|\tau_I/\rho_L g m|$ the following simplified relation is obtained:

$$\frac{f_I}{f_s} = 1 + 1400F. \tag{2}$$

3. FILM HEIGHTS FOR HORIZONTAL FLOWS

Because the film is distributed asymmetrically around the pipe circumference, the relations describing average film heights for horizontal flows can be different from those obtained for vertical flows. We define the local time averaged film height as h and the average height around the pipe perimeter as $\langle h \rangle = m$. The approach taken in this paper is to assume that h is given by[11].

Thus, for low local liquid film flow rates

$$\frac{4\Gamma}{\nu_L} = 2h^2(\tau_c/\rho_L)/\nu_L{}^2, \tag{17}$$

where Γ is the local volumetric flow rate per unit perimeter. If [17] is integrated around the pipe perimeter the following relation is obtained:

$$\mathrm{Re}_{LF} = 2\langle h^{+2} \rangle, \tag{18}$$

where $\langle h^{+2} \rangle$ is the average value of h^{+2} around the pipe perimeter. The nondimensional spatially averaged film thickness, m^+, is defined as

$$m^+ = \langle h \rangle \langle (\tau_C/\rho_L)^{1/2} \rangle / \nu_L, \tag{19}$$

with $\langle h \rangle = m$. It is evident from the above definition that $\langle h^{+2} \rangle \neq m^{+2}$. For very small Re_{LF} the film would be very thin so τ_I might not be very different from that for a smooth film, and $\tau_I \simeq \tau_C$. Therefore $\langle h^{+2} \rangle = \langle h^2 \rangle (\tau_s/\rho_L)^{1/2} \nu_L$ and $m^+ = m(\tau_s/\rho_L)^{1/2}/\nu_L$. For $\mathrm{Re}_{LF} \to 0$,

$$\mathrm{Re}_{LF} \simeq 2\frac{\langle h^2 \rangle}{m^2} m^{+2}. \tag{20}$$

The term $\langle h^2 \rangle / m^2$ represents a correction for asymmetries in horizontal flow for laminar films.

For high Re_{LF} the local film height is assumed to be related to the local film flow rate by the relation

$$h^+ = 0.0379\left(\frac{4\Gamma}{\nu_L}\right)^{0.9}. \tag{21}$$

The integration of [21] around the pipe perimeter gives

$$m^+ = \frac{0.0379\ \mathrm{Re}_{LF}^{0.9}}{\langle h^{+1.1}\rangle^{.9}/m^+}. \qquad [22]$$

The term $\langle h^{+1.1}\rangle^{.9}/m^+$ is the correction for film asymmetry for large Re_{LF}.

4. CORRELATION OF FRICTION LOSS MEASUREMENTS

Pressure drops in the fully developed region are accompanied by changes of gas density. Therefore, these measurements are associated with accelerations of the gas and entrained liquid droplets as well as with frictional losses. If it is assumed that the flow is isothermal and that the relative velocity, or slip ratio, of the drops and the gas, $K = U_G/U_D$, does not change over the length of pipe, L, for which measurements are being made, the following equation can be derived:

$$\langle f_I\rangle = \frac{\pi^2(d_t-2m)^5 p_2\rho_{G2}}{4L\ 16W_G^2}\left[\left(\frac{p_1}{p_2}\right)^2 - 1\right] - \frac{(d_t-2m)}{2L}\left(1+\frac{1}{K}\frac{W_{LE}}{W_G}\right)\ln\left(\frac{p_1}{p_2}\right)$$
$$-\frac{(d_t-2m)}{2L}\frac{1}{K}\left(\frac{W_L}{W_G}\right)(E_2-E_1). \qquad [23]$$

The average friction factor is calculated from [23] using measurements of changes of pressure and entrainment from upstream values of p_1 and E_1 to downstream values of p_2 and E_2 over the pipe length L.

The frictional pressure gradient is related to f_I through the equation

$$\left|\frac{\mathrm{d}p}{\mathrm{d}z}\right|_f = \frac{2f_I}{d_t}\rho_G U_{SG}^2 \frac{d_t^5}{(d_t-2m)^5}. \qquad [24]$$

If a modified interfacial friction factor is defined as

$$f_I^* = f_I\frac{d_t^5}{(d_t-2m)^5}, \qquad [25]$$

then the Martinelli group

$$\phi_G^2 = \left(\frac{\mathrm{d}p}{\mathrm{d}x}\right)_f \Big/ \left(\frac{\mathrm{d}p}{\mathrm{d}x}\right)_{Gf} \qquad [26]$$

is given as

$$\phi_G^2 = f_I^*/f_s. \qquad [27]$$

Here f_s is defined by [15].

From the discussion in section 2(b) one might expect that measurements of f_I could be correlated with the relation

$$\frac{f_I}{f_s} \sim F_H, \qquad [28]$$

with

$$F_H = \frac{\gamma_H(\mathrm{Re}_{LF})}{\mathrm{Re}_G^{0.90}}\left(\frac{\nu_L}{\nu_G}\right)\sqrt{\frac{\rho_L}{\rho_G}}. \qquad [29]$$

From [24] and [25] it is seen that measurements of m/d_t would be needed in addition to f_i/f_s in order to evaluate the frictional pressure loss.

However, if m/d_t varies as F_H a more convenient method for correlating data for the frictional pressure loss would be to plot ϕ_G^2 vs F_H.

5. DESCRIPTION OF EXPERIMENTS

The experiments were conducted in a flow loop described in theses by Dallman (1978) and Laurinat (1982). Pressure drop and film height measurements were made in plexiglas test sections with inside diameters of 2.54 cm and 5.08 cm, located, respectively, 550 and 300 pipe dia. from the entry. Tests conducted by Dallman (1978) indicate the flow should be fully developed at these locations. He compared film thickness, pressure drop and entrainment measurements taken 200 and 550 dia. downstream from his two phase mixer. With the possible exception of the entrainment for high liquid flow rates, he found no differences between measurements at these two locations other than those caused by changes in gas velocities and pressures.

The pressure drop was determined using the liquid phase. In order to avoid the presence of air in the pressure lines, which can cause errors of 100% or more (Hewitt & Hall–Taylor 1970), a small purging flow to each tap of less than 0.1% of the total liquid flow was necessary. Dallman (1978) used a liquid–liquid manometer to obtain his pressure drop measurements in the 2.54 cm pipe. Laurinat (1982), in his studies with the 5.08 cm pipe, found that a diaphragm-type differential pressure transmitter gave more reliable pressure drop measurements than a liquid–liquid manometer. Consequently he repeated the pressure drop measurements of Dallman so that a more consistent comparison could be made between the studies in the two pipe sizes. All of the pressure drop measurements reported in this paper are from the thesis by Laurinat.

The film height at a given circumferential location was determined by measuring the conductance of water between two parallel wires or, for very thin films, between two flush mounted plates. Both gave a linear response over the range of heights for which they were used.

The parallel wire probes were made by inserting chromel wires, sized 24 A.W.G. (0.511 mm dia.), through the wall of the test section 2.54 mm apart. The wires were aligned perpendicular to the direction of flow. Eight probes, spaced at 45° intervals around the pipe circumference, were used.

The flush mounted probes were installed in plugs that could be inserted in the test section interchangeably with plugs containing the parallel wire probes. The electrodes were formed by epoxying 1.59 mm by 11.75 mm stainless steel bars flush with the wall. The electrodes were separated by a 1.59 mm insulating gap oriented perpendicular to the direction of flow.

The average height, m, was calculated using a perimeter weighted average, the thickness measured at each probe being considered to cover 12.5% of the pipe perimeter.

Flowrates of the wall film were measured by withdrawing it through a section of porous pipe wall. Details regarding this technique may be found in theses by Dallman (1978) and Laurinat (1982).

6. RESULTS

The range of gas and liquid flow rates over which measurements were obtained is shown in figure 1. The conditions for the existence of annular flow defined by Dallman (1978) for the 2.54 cm pipe and by Laurinat (1982) for the 5.08 cm pipe are indicated by the dashed and dotted curves. It is to be noted that the transitions from slug to annular flow and from stratified to annular flow are different from those given by Mandhane *et al.* (1974).

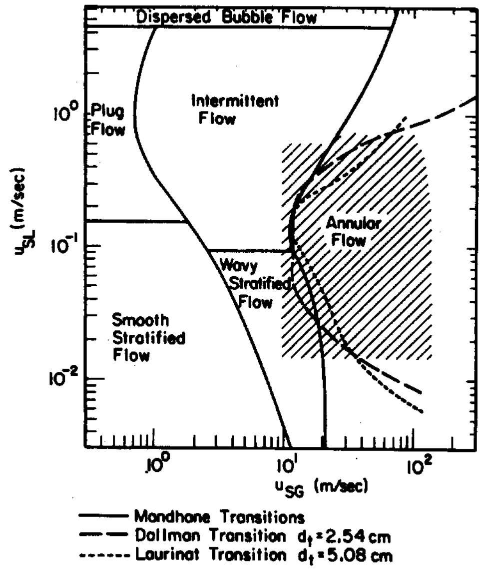

Figure 1. Range of variables studied and region of annular flow defined by Dallman and Laurinat. Hatched area is the region over which ΔP measurements were made.

Dallman and Laurinat defined slug flow as a flow where the absolute pressure varied by about 1 cm of mercury during a pulse. It is quite possible that these pulses were associated with large amplitude waves or aggregated droplets which did not quite bridge the whole pipe cross section. If this is the case, then the suggested gas velocities for the transitions to annular flow in figure 1 define a transition from some other pattern than slug flow. The transition from annular to stratified flow is defined to occur when the liquid layer does not form a continuous film about the perimeter of the pipe. It appears as a break up of the film on the top portion of the pipe into rivulets.

Measurements of the average film height for the 2.54 cm pipe at 500 pipe dia. from the inlet are shown in figure 2 as a plot of m^+ vs the film Reynolds number, defined as $\mathrm{Re}_{LF} = 4W_{LF}/\mu_L P$. The liquid film flow rates were determined by withdrawing the wall film through a porous section of pipe wall or by measuring droplet fluxes in the air. The characteristic stress used to evaluate m^+ is defined by [8]. The interfacial stress was calculated from experimental measurements of pressure drop using [23] and [14]. The wall

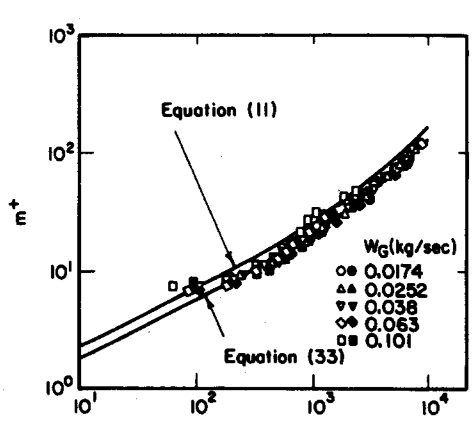

Figure 2. Correlation of film height measurements taken in the 2.54 cm pipe. Open points are for $\rho_G = 1.34\ \mathrm{kg/m^3}$; the closed points, for $\rho_G = 2.73\ \mathrm{kg/m^3}$.

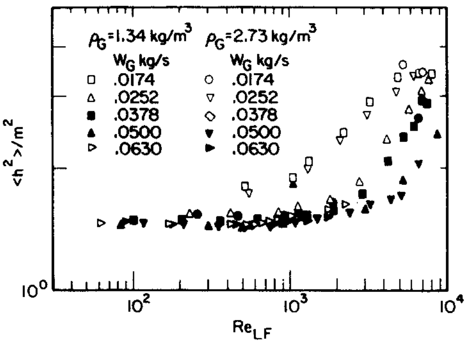

Figure 3. Measurements of the small Reynolds number correction for the film height relation taken in the 2.54 cm pipe.

shear stress was obtained from a force balance for the liquid film for fully developed flow,

$$\left|\frac{\mathrm{d}p}{\mathrm{d}z}\right|_f = \frac{4\tau_w}{d_t} = \frac{4\tau_I}{(d_t - 2m)}. \tag{30}$$

It can be seen that the equation developed by Henstock & Hanratty for vertical flows consistently overpredicts m^+. This can be explained because of the asymmetry of the film in horizontal flow.

It is shown in section 3 that the film height relation at low Reynolds numbers can be corrected for asymmetries by using the factor $\langle h^2\rangle^{1/2}/m$. Measurements of this factor in the 2.54 cm pipe, presented in a thesis by Dallman (1978), are shown in figure 3. At small Re_{LF} it is seen that $\langle h^2\rangle^{1/2}/m$ approaches 1.4. By substituting this value in (20) the following equation is obtained for $\mathrm{Re}_{LF}\to 0$:

$$m^+ = 0.50\,\mathrm{Re}_{LF}^{1/2}. \tag{31}$$

Values of $\langle h^{+1.1}\rangle^{0.9}/m^+$, obtained by Dallman (1978) by assuming that the relation between the local τ_I and h/d_t is the same as the empirically determined relation between $\langle\tau_I\rangle$ and m/d_t, are plotted in figure 4. For large Re_{LF} this factor approaches a value of 1.38. If this is substituted into [22] the following equation is obtained:

$$m^+ = 0.028\,\mathrm{Re}_{LF}^{0.90}. \tag{32}$$

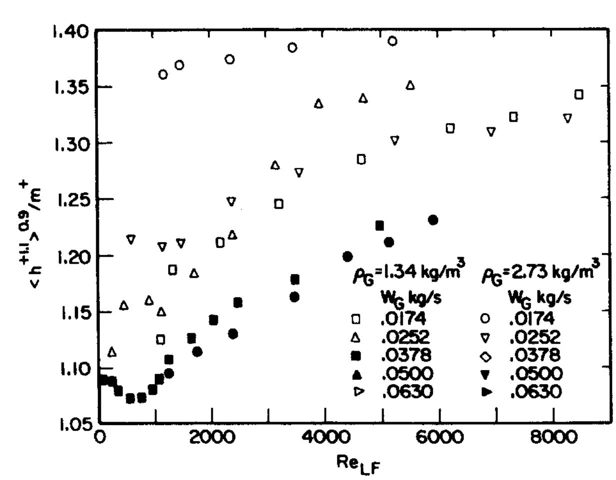

Figure 4. Measurements of the large Reynolds number correction for the film height relation taken in the 2.54 cm pipe.

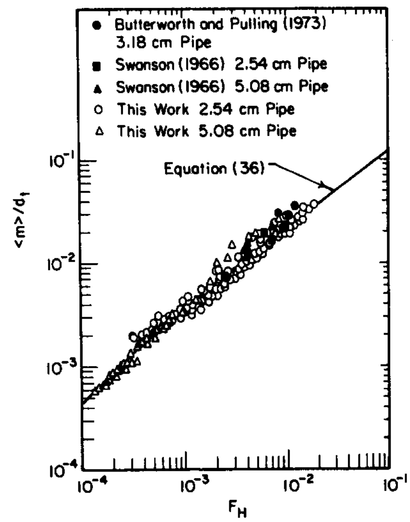

Figure 5. Film holdup correlation.

The m^+ measurements shown in figure 2 are fit reasonably well by the following combination of asymptotic solutions close to those given by [31] and [32]:

$$m^+ = \gamma_H(\mathrm{Re}) = [(0.566\ \mathrm{Re}_{LF}^{0.5})^{2.5} + (0.0303\ \mathrm{Re}_{LF}^{0.9})^{2.5}]^{0.4}. \quad [33]$$

Measurements obtained in the 2.54 cm pipe at 200 dia. from the entrance and in the 5.08 cm pipe at 300 dia. from the entrance are also described by [33]. [See figure 60 in the thesis by Dallman (1978) and figures 26 and 27 in the thesis by Laurinat (1982)].

As has been shown by Henstock & Hanratty (1976) the correlation for film height [33] can be rearranged in the following more convenient form:

$$\frac{m}{d_t} = \frac{6.59F_H}{(f_i/f_s)^{1/2}} \frac{\left(1 - \dfrac{2m}{d_t}\right)^{1/2}}{\left(1 - \dfrac{2m}{3d_t}\right)^{1/2}} \quad [34]$$

Here F_H is the flow factor defined by [29] and [33]. Evaluation of m from [34] requires the determination of (f_i/f_s). However, since this appears to the 1/2 power, the accuracy of this

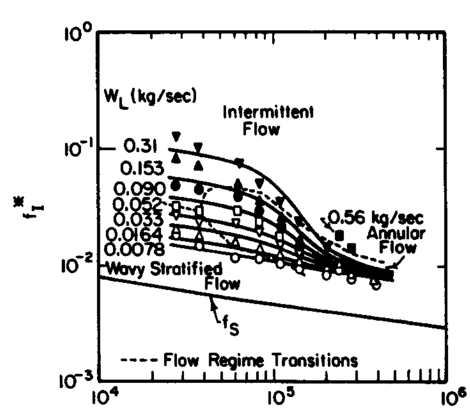

Figure 6. Correlation of frictional pressure drop measurements in the 2.54 cm pipe, calculated using $K = 2$.

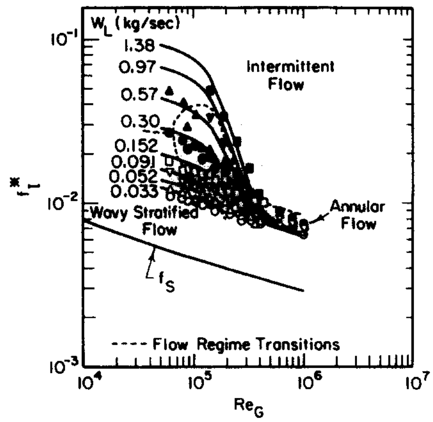

Figure 7. Correlation of frictional pressure drop measurements in the 5.08 cm pipe, calculated using $K = 2$.

determination need not be high. Consequently, we explored the case where (f_I/f_s) is a function of F_H. It follows from [34] that under these circumstances m/d_t should also be a function of F_H. Such a plot is presented in figure 5. Measurements obtained by Butterworth (1973) in a 3.18 cm pipe and by Swanson (1966) in a 2.54 cm pipe are also included. Approximate agreement is obtained between measurements in the 2.54 cm pipe and the 5.08 cm pipe.

A correlation of the measurements in figure 5 with F_H may be derived by combining an approximation to [34],

$$\frac{m}{d_t} = \frac{6.59F_H}{(f_I/f_s)^{1/2}} \tag{35}$$

with an asymptotic correlation of f_I/f_s for pressure drop measurements at high gas velocities [40]:

$$\frac{m}{d_t} = \frac{6.59F_H}{[(2.3)^5 + (90F_H)^5]^{0.2}} \tag{36}$$

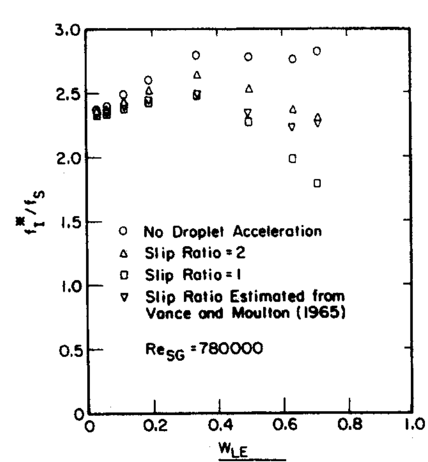

Figure 8. Effect of assumption regarding droplet acceleration on the calculation of f_I^* from measurements in the 5.08 cm pipe.

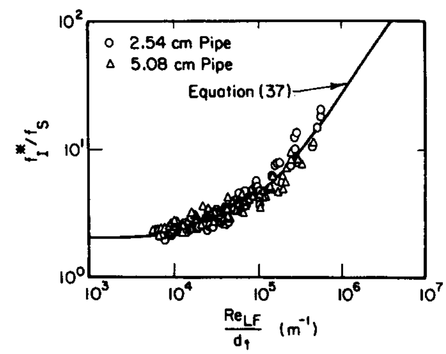

Figure 9. Correlation of the friction factor with film properties.

Although [40] does not correlate pressure drops at low gas velocities, it approximates the friction factor ratio well enough to be used to predict m/d_t.

The frictional pressure drop measurements are plotted as

$$f_I^* = \frac{d_t}{2}\left|\frac{dP}{dz}\right|_f \Big/ \rho_G U_{SG}^2 \qquad [37]$$

vs a gas Reynolds number, defined using the tube diameter, in figures 6 and 7. It is noted that for all liquid flow rates, for which annular flow exists, the friction factor appears to approach an asymptotic value at large Re_G

$$f_I^* \simeq 2.3 f_s. \qquad [38]$$

This same behavior had been previously noted by Aziz & Govier. (See figure 10.5 of Govier & Aziz 1972.) This lower asymptote to the ratio f_I^*/f_s appears to be associated with the observation that, even at high gas velocities, an annular film with mass flow rate W_{LFC} remains on the walls of the pipe.

In calculating f_I^* from pressure drop measurements using [23], a value of $K = 2$ was used. Figure 8 shows that the choice of K can have an effect at high gas velocities for a high amount of entrained liquid. Measurements of the slip ratio have been obtained by Vance & Moulton (1965). Calculated values of the friction factor using their measurements of K are shown to be in good agreement with calculated values using $K = 2$.

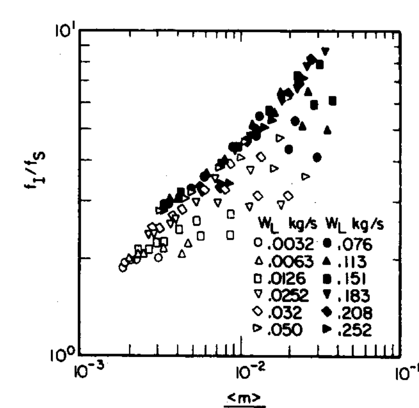

Figure 10. Comparison of the friction factor ratio with film holdup for measurements taken in the 2.54 cm pipe 500 dia. from the entry.

The results in figures 6 and 7 indicate that the friction factor is a function only of film properties and not of the amount of liquid entrained. The sharp decrease in f_I^*/f_s with the initiation of annular flow can be interpreted as due to a sharp decrease in the flowrate of the liquid film due to atomization. This is more clearly illustrated in figure 9 where the f_I^*/f_s values shown in figures 6 and 7 are plotted against the film Reynolds number. It is noted that the results for a given pipeline are correlated quite well. However the correlation $f_I^*/f_s = f(\mathrm{Re}_{LF})$ does not predict the effect of pipe diameter; i.e. at the same Re_{LF} the values of f_I^*/f_s are smaller for the larger diameter pipe. It is noted that the measurement for both pipelines can be brought together approximately by using Re_{LF}/d_t as the abscissa. An empirical equation that appears to fit the measurements is

$$\frac{f_I^*}{f_s} = 2 + 2.5 \times 10^{-5}\frac{\mathrm{Re}_{LF}}{d_t}, \qquad [39]$$

where d_t is in meters.

Previous work for vertical gas–liquid annular flows suggests that the friction factors for different pipe sizes should scale as the ratio of the film thickness to the pipe diameter. Such a correlation is explored in figure 10 for data in the annular flow region for the 2.54 cm pipe at a distance of 500 dia. from the entry. At high gas velocities (greater than 30 m/sec) where the film is more uniformly distributed the friction factor approaches an asymptotic relation. However at small gas velocities (less than 30 m/sec) where the film is highly asymmetric, resembling a stratified flow with a highly agitated interface, the friction factor ratio appears to be mainly a function of the liquid Reynolds number. A similar plot of measurements in the 5.08 cm pipe presented in the thesis by Laurinat shows more scatter.

From the type of results shown in figure 10 and the discussion presented in section 2(b), it might be expected that the friction factor ratio for annular flows should be a function of the flow factor F_H defined by [29] and [33]. Such a plot is made in figure 11 for data taken in both pipes at gas velocities greater than 30 m/sec. Measurements made by Butterworth (1973) in a 3.18 cm pipe and by Swanson (1966) in 2.54 cm and 5.08 cm pipes are also shown in figure 11. These are in reasonable agreement with our measurements in a 2.54 cm pipe.

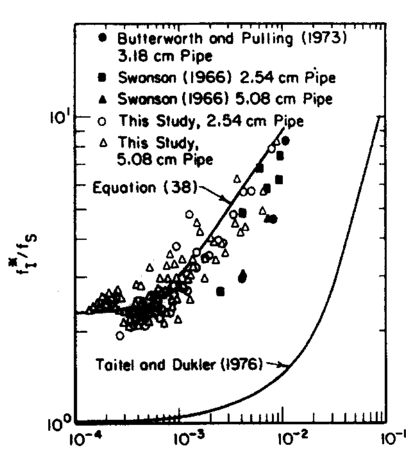

Figure 11. Friction factor relation for gas velocities greater than 30 m/sec. The Taitel–Dukler relation is for stratified flow of a laminar liquid and a turbulent gas.

The following equation roughly fits the measurements in figure 11:

$$\frac{f_I^*}{f_s} = [(2.3)^5 + 90F_H^{0.5})^5]^{0.2}. \tag{40}$$

It is evident that this correlation is not completely satisfactory. Further improvement will require that account be taken of the asymmetry of the film in addition to the flow factor F_H. The inclusion of methods for correcting for asymmetry might also make it possible to develop a correlation which includes annular flow data for low gas velocities.

For cases in which there is no entrainment the flow factor F_H is the same as the Martinelli flow parameter. Recent work by Taitel & Dukler (1976) suggests that for stratified flows with smooth interfaces pressure drop data should be correlated as f_I^*/f_s vs F_H. Their correlation, presented for comparison in figure 11, indicates much lower pressure drops than are observed in annular flows or stratified flows with highly agitated interfaces. These results clearly show the difficulty of trying to use a single Martinelli plot to correlate results for all flow regimes.

The transition from a smooth stratified flow through a wavy stratified flow to an annular flow would require some method for defining the transition between the two curves shown in figure 11. The solution of this problem is a key to obtaining a better correlation of the annular flow measurements than presented in this paper, since annular flows at low gas and liquid velocities might be considered part of the transitional region.

2. CONCLUSIONS

Film height measurements for both the 2.54 cm and 5.08 cm pipelines are described by the relation

$$m^+ = [(0.566\ \mathrm{Re}_{LF}^{0.5})^{2.5} + (0.0303\ \mathrm{Re}_{LF}^{0.9})^{2.5}]^{0.4}. \tag{33}$$

The values of m^+ predicted for a given value of Re_{LF} are less than what is observed for vertical flows. This can be explained if account is taken of the asymmetry of the film in horizontal flows.

As indicated by [34] the above equation requires values of f_i/f_s in order to evaluate m/d_t for a given liquid film Reynolds number. A more convenient, but less accurate, relation is

$$\frac{m}{d_t} = \frac{6.59F_H}{[2.3^5 + (90F_H)^5]^{0.2}}, \tag{36}$$

with F_H defined by [29].

The pressure drop results from frictional losses and from the acceleration of the gas and the droplets due to the change in gas velocity with decreasing pressure. The assumption of $K = U_G/U_D = 2$ gives estimates of the contribution of droplet acceleration consistent with what would be predicted from measurements by Vance & Moulton (1965). The frictional pressure loss is calculated from the equation

$$\left|\frac{\mathrm{d}p}{\mathrm{d}z}\right|_f = \frac{2f_I^*}{d_t}\rho_G U_{SG}^2. \tag{24}$$

It is found that f_I^* depends entirely on properties of the flowing wall film; i.e. it is independent of the concentration of droplets in the gas flow. For a given pipe diameter this can be expressed by the relation

$$\frac{f_I^*}{f_s} = 2 + f(\mathrm{Re}_{LF}). \qquad [41]$$

For a fixed liquid flow rate, [41] predicts an increase in f_I^*/f_s with decreasing gas velocity because of the decrease in the flow rate of entrained liquid.

Values of f_I^*/f_s at a given Re_{LF} decrease with increasing pipe diameter. The results for the two pipe diameters investigated can be represented by the empirical relation

$$\frac{f_I^*}{f_s} = 2 + 2.5 \times 10^{-5}\frac{\mathrm{Re}_{LF}}{d_t}, \qquad [39]$$

where d_t is in meters. An explanation of this strong effect of pipe diameter is explored which assumes that f_I^*/f_s varies with m/d_t. This is only partially successful, as can be seen by comparing [40] with measurements.

The best correlation of the measurements of film height and frictional pressure loss obtained in the experiments is therefore given by [36], [24] and [39]. The use of these equations requires a knowledge of the entrainment in order that Re_{LF} be specified for given gas and liquid flow rates. Equation [1] may be used for this purpose. The curves shown in figures 6 and 7 were calculated in this manner.

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NOTATION

- d_t tube diameter
- E fraction entrained $= W_{LF}/W_L$
- f_I friction factor for a rough interface $2\tau_I/\rho_G U_G^2$
- f_s friction factor for a smooth surface $2\tau_s/\rho_G U_G^2$
- f_I^* modified interfacial friction factor $= f_I d_t^5/(d_t - 2m)^5$
- F flow factor for vertical pipes defined by [13]
- F_H flow factor for horizontal pipes defined by [29]
- g acceleration of gravity
- $\mathbf{g}$ gravity and pressure gradient factor defined by [4]–[6]
- h local film height
- K relative velocity of the drops and the gas $= U_G/U_D$
- L length of section used to measure pressure drop
- m average height of the film around the pipe circumference $= \langle h \rangle$
- m^+ dimensionless average film height $= mv^*/\nu_L$
- p pressure
- $\left|\frac{dp}{dz}\right|$ absolute value of the pressure gradient
- $\left|\frac{dp}{dz}\right|_f$ contribution to the pressure gradient due to friction
- $\left|\frac{dp}{dz}\right|_s$ frictional pressure gradient if gas were flowing alone in the pipe
- P perimeter of the pipe
- Q volumetric flow of the liquid in the film

r	radial location in the pipe
R	pipe radius
Re_G	gas phase Reynolds number $= [(d_t - m)U_G/\nu_G]$
Re_{LF}	liquid film Reynolds number $= 4Q/P\nu_L$
U_G	gas velocity
U_D	droplet velocity
U_{GS}	superficial gas velocity
v^*	friction velocity $= (\tau_c/\rho_L)^{1/2}$
W_L	mass flow of the liquid
W_{LF}	mass flow of the liquid film
W_{LE}	mass flow of the entrained liquid
W_{LFC}	critical flowrate of the liquid film
W_G	mass flow rate of the gas

Greek letters

αm^+	stress distribution factor for the film $= -\tilde{g}m/v^{*2}$
$\gamma(\mathrm{Re}_{LF})$	dependency of m^+ on liquid film Reynolds number for vertical pipes given by [11]
$\gamma_H(\mathrm{Re}_{LF})$	dependency of m^+ on liquid film Reynolds number for horizontal pipes given by [33]
Γ	local liquid film mass flow rate per unit perimeter length
μ	viscosity
ϕ_G^2	Martinelli group $= \left\|\frac{\mathrm{d}p}{\mathrm{d}z}\right\|_f \Big/ \left\|\frac{\mathrm{d}p}{\mathrm{d}z}\right\|_{Gf} = f_I^*/f_I$
$\gamma_H(\mathrm{Re}_{LF})$	dependency of m^+ on liquid film Reynolds number for horizontal pipes given by [33]
ν	kinematic viscosity
τ	shear stress
τ_w	shear stress at the wall
τ_I	shear stress at the interface
τ_c	characteristic stress for the liquid film defined by [7] and [8]
τ_s	shear stress at a smooth wall

Subscripts

G	refers to the gas
L	refers to the liquid
1	location at upstream pressure tap
2	location at downstream pressure tap

Brackets

$\langle\ \rangle$	signifies an average value around the circumference
$\|\ \|$	signifies an absolute value

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